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An Analysis of Students' Conceptual Understanding in the Pythagorean Theorem Material

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ABSTRACT

Conceptual understanding is a fundamental skill and one of the primary goals of mathematics learning, where students are expected to comprehend concepts, explain relationships between concepts, and apply them in mathematical problem-solving processes. However, observations reveal that students generally exhibit relatively low levels of conceptual understanding when solving mathematical problems, mainly due to insufficient mastery of prerequisite materials. The purpose of this study is to analyze students' conceptual understanding of the Pythagorean Theorem based on the conceptual understanding indicators proposed by the National Research Council. The research method used is qualitative with a descriptive approach, specifically a hermeneutic phenomenological study. Qualitative data were obtained from document analysis, test results, and interviews involving 29 junior high school students. The findings based on five test items representing the conceptual understanding indicators show that the most mastered indicator by students is the ability to apply and use the concept appropriately. Conversely, the least mastered indicator is the ability to classify objects based on mathematical concepts.

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INTRODUCTION

Conceptual understanding is a fundamental skill in mathematics learning that students must master [1]. Students who have achieved conceptual understanding are considered to have gone through the processes of memorizing and analyzing, enabling them to grasp the meaning of concepts. This is demonstrated through their ability to construct messages both orally and in writing, as well as through graphical representations [2]. The mathematical conceptual ability is one of the objectives of mathematics subjects, which can be emphasized through in-depth learning related to critical thinking skills and reflective abilities in solving mathematical problems.

However, in reality, the results of school observations show that there are still students with low conceptual understanding when solving mathematical problems. Students are often unable to apply formulas because they do not understand the concepts behind them, leading to frequent mistakes in substituting values. This is also due to the generally low basic mathematical skills of the students. This is consistent with research by [3], which found that the difficulties students face in solving mathematical problems are often related to problems involving word problems or real-life contexts. These difficulties result in errors such as creating illustrations, identifying known and unknown information, selecting the correct formula, performing calculations or operations, and drawing conclusions [3].

One of the mathematics topics that requires strong conceptual understanding is the

Pythagorean Theorem. The Pythagorean Theorem demands that students be both knowledgeable and careful when solving problems related to it [4]. In research conducted by [5], it was found that students struggled to understand the concept of problems that required them to write the relationship between sides p, q, and r based on the Pythagorean Theorem. Students also had difficulties identifying the hypotenuse and applying the formula to determine the other two sides when the hypotenuse value was given. In reality, students still struggle to distinguish between formulas used to determine the length of the hypotenuse and the other sides. This difficulty often stems from students' lack of understanding of the Pythagorean concept itself and from not having discovered the proof of the theorem on their own, which leads to limited comprehension.

Based on interview results with a mathematics teacher, it was found that many students still lack understanding of the Pythagorean Theorem, which affects their ability to solve more advanced problems involving its application. Students also struggle to determine the length of the hypotenuse or other sides due to not understanding how to use the formula correctly. As a result, they are unable to solve word problems related to the Pythagorean Theorem, including drawing diagrams and completing the solutions.

According to research by [6], students' difficulties in understanding stem from their inability to internalize the concept, which prevents them from solving problems in different contexts. Their failure to extract implicit information and create visual representations makes students tend to avoid word problems or problems with long questions, thus further hampering students' mathematical abilities, especially their conceptual understanding abilities. Therefore, this study aims to identify and analyze students' conceptual understanding abilities in the Pythagorean Theorem material based on conceptual understanding ability indicators.

METHODS

The method used in this study is qualitative with a descriptive approach. This method examines phenomena related to the experiences of research subjects in a holistic manner, which are then interpreted in an actual, systematic, and reliable way [7]. The aim of using this descriptive qualitative method is to accurately and systematically describe facts and characteristics of the subjects being studied. The qualitative data obtained from document analysis and written test results are explained by summarizing and concluding the findings of the research.

The purpose of this study is to analyze students' conceptual understanding in learning the topic of the Pythagorean Theorem. According to the National Research Council, as cited in [8], there are five indicators of students' mathematical conceptual understanding, namely: (1) restating the learned concept; (2) classifying objects based on mathematical concepts; (3) correctly applying and implementing the concept; (4) presenting concepts and solving problems using various mathematical representations.

The subjects of this study were 29 eighth-grade junior high school students. The instrument used was an essay test consisting of five questions that had gone through a validation process. The data analysis technique used included: (1) data reduction, by summarizing and focusing on essential points to concentrate attention on the data obtained from observations; (2) data presentation, which is the process of organizing relevant data to obtain information with specific meaning; and (3) concluding, supported by valid and consistent evidence.

A conclusion drawing was carried out based on the analysis of students' mathematical conceptual understanding of the test results. The data validation method used was triangulation. The triangulation involved collecting data through interviews, direct observations, and surveys. The first

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step taken was to give students essay questions related to solving Pythagorean Theorem problems. Then, the students' responses were analyzed and matched with the indicators of conceptual understanding. The final step was to confirm the analyzed responses by interviewing selected students based on their answers and how they arrived at those answers, in order to conclude their level of conceptual understanding.

RESULTS AND DISCUSSION

The results obtained from this study, with a sample of 29 students who were given a test to identify their conceptual understanding skills, show that five students had mastered all the indicators of understanding, 10 students had mastered four indicators, 11 students had mastered three indicators, two students had mastered two indicators, and one student had learned only one indicator. The following is a description and discussion of students' mathematical conceptual understanding of the Pythagorean Theorem:

1. Restating the Concept that Has been Learned

In this indicator, the problem given involves determining the Pythagorean Theorem formula from a presented triangle diagram, with each side labeled using specific symbols.

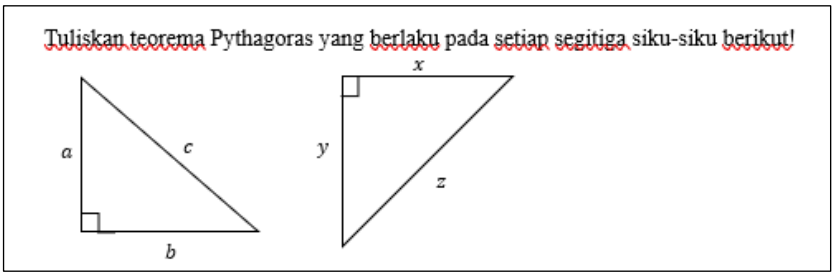

Figure 1. Question for the Indicator of Restating the Concept that Has been Learned

Based on Figure 1, the question is presented in the form of a triangle diagram, and students are asked to write the applicable Pythagorean Theorem for the given triangle. Students are expected to understand the Pythagorean Theorem formula in order to determine the length of one side of a right triangle or to state the general formula of the theorem. Based on the analysis of students' answers, it was found that nearly all students answered correctly — 23 students — while six students were still unable to restate the learned concept. The common errors in students' responses included incorrectly stating the formula in the second diagram and reversing the formula when finding the hypotenuse. This was also previously stated by [9], who found that conceptual errors made by students were due to a lack of understanding of the question, which led them to construct the appropriate formula incorrectly. The following are several images of students' answers.

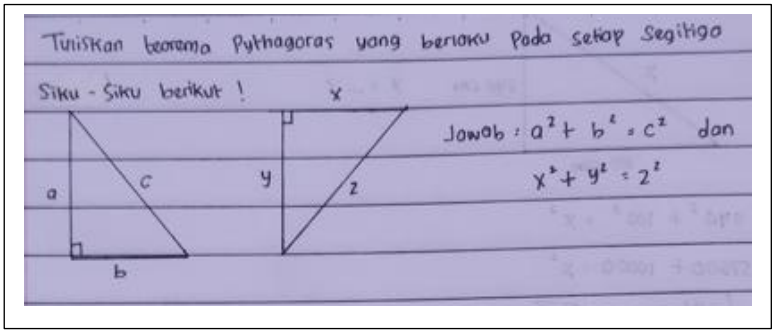

Figure 2. Example of a Student's Answer that Meets the First Indicator

Based on Figure 2, it can be seen that the student is able to correctly write the Pythagorean Theorem formula using the given triangle diagram. From the interview results, it was found that the student understood how to identify the diagram and formulate the Pythagorean Theorem accurately and correctly. The following is an example of a student's answer that does not meet the first indicator.

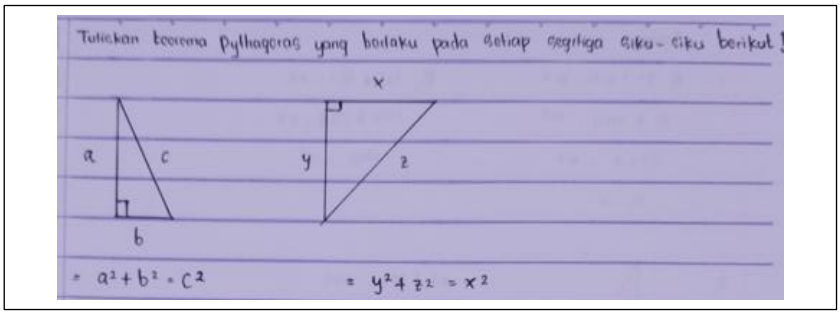

Figure 3. Example of a Student's Answer that Does Not Meet the First Indicator

Based on Figure 3, it can be seen that the student was able to correctly write the Pythagorean Theorem for the diagram on the left, but failed to do so for the diagram on the right, which means the error occurred after the position of the triangle was slightly altered. This type of response was found among all students who answered the first indicator question incorrectly. From the student interviews, it was revealed that the students made errors in determining the Pythagorean Theorem formula for the second diagram due to the changed orientation, which was different from the first diagram and from the typical right triangle images they were familiar with. This finding is also supported by [10], who noted that in this type of problem, students experience conceptual errors due to their inability to choose the correct and appropriate formula.

2. Classifying Objects Based on Mathematical Concepts

In this indicator, the given question involves identifying examples and non-examples of right triangles using the Pythagorean Theorem, allowing students also to recognize Pythagorean triple numbers.

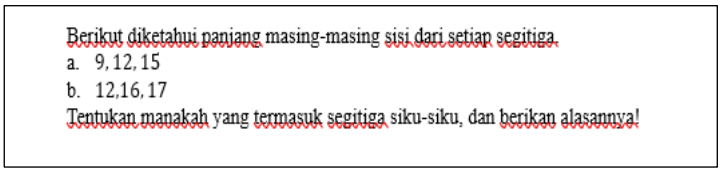

Figure 4. Question for the Indicator of Classifying Objects Based on Mathematical Concepts

Based on Figure 4, the question presented requires students to identify Pythagorean Triples by determining examples and non-examples using the Pythagorean Theorem. The analysis of students' responses to this question showed that 18 students answered correctly, but only 5 of them provided accurate reasoning. Meanwhile, 11 students were unable to answer correctly — most of them only verified one triple (the correct one), and almost none gave a reason for their answers. This was also found by [10], who noted that in this indicator, most students encountered conceptual, procedural, and technical difficulties, as seen from their responses, which showed an inability to apply the Pythagorean Theorem and to provide examples of it. The following are several images of students' answers to the second indicator question.

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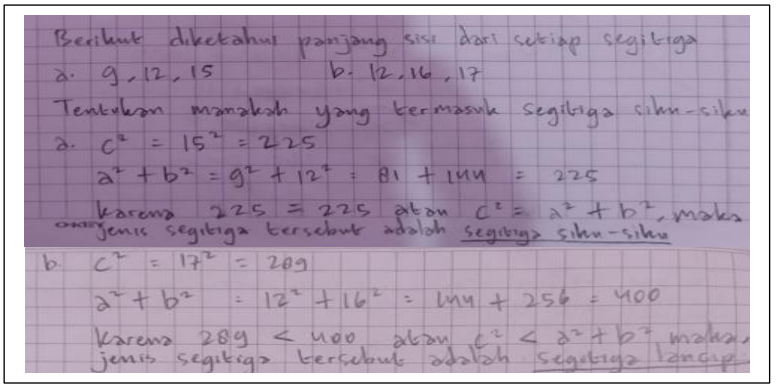

Figure 5. Example of a Student's Answer that Meets the Second Indicator

Based on Figure 5, it was found that the student was able to determine Pythagorean Triples using the concept of the Pythagorean Theorem and was also able to provide reasoning and draw accurate conclusions. From the interview conducted, it was revealed that the student could correctly identify both examples and non-examples of Pythagorean Triples. The following is an example of a student's answer that does not meet the second indicator.

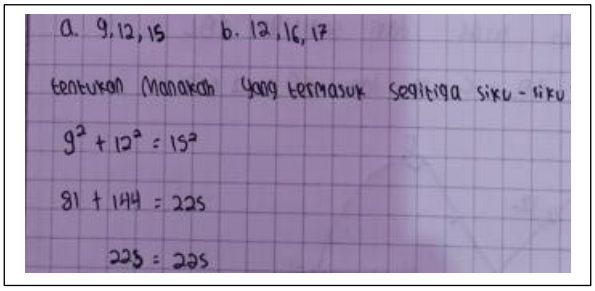

Figure 6. Example of a Student's Answer that Does Not Meet the Second Indicator

Based on Figure 6, it was found that the student was able to identify a method for testing a Pythagorean triple, but did not complete the second part of the problem. The student tended to stop working on it after finding one solution and did not provide a summary or conclusion of their findings. Many students did not write a conclusion or explanation, but only presented the verification of the Pythagorean triple, resulting in poorly structured answers. Similar findings were reported in studies by [11] and [12], which showed that students still made technical errors during operational processes. Based on interview results, it was revealed that students only attempted trial and error using the Pythagorean Theorem until they arrived at a correct answer. After doing so, they did not continue to try the next question or provide adequate conclusions and reasoning.

3. Applying and Implementing the Concept Correctly

In this indicator, the given question involves applying the concept of the Pythagorean Theorem by determining one side of a right triangle when the other sides are known.

Suatu segitiga siku-siku memiliki panjang hipotenusa 34 cm, dengan salah satu sisi siku-siku adalah 16 cm, tentukan panjang sisi siku-siku lainnya!

Figure 7. Question for the Indicator of Correctly Applying and Implementing the Concept

Based on Figure 7, the given question requires students to apply the concept of the Pythagorean Theorem to determine the length of one side of a triangle, and students must also understand terms related to the theorem, such as hypotenuse and others. Based on the analysis of students' answers, most students were able to apply the Pythagorean Theorem correctly; however, two students still did not fully understand the concept and made errors in finding the correct answer. This is consistent with the study by [13], which found that common errors include conceptual, procedural, and technical mistakes. The following are several examples of students' answers related to this third indicator.

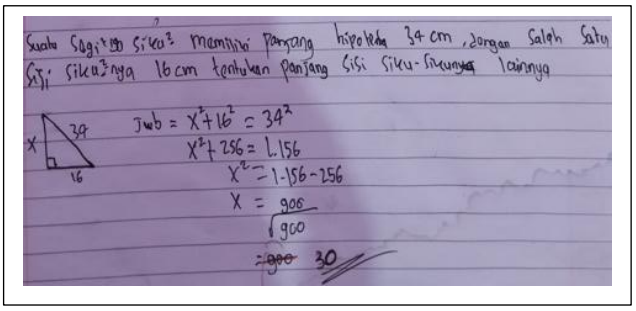

Figure 8. Example of a Student's Answer that Meets the Third Indicator

Based on Figure 8, it was found that the student was able to apply the Pythagorean Theorem correctly. First, the student wrote the general formula of the Pythagorean Theorem, then modeled it according to the problem's requirements and substituted the known values. From the interview results, it was revealed that the student understood the terms related to the Pythagorean Theorem and was able to manipulate the required formula to solve the problem and arrive at the correct answer. The following is an example of a student's answer that does not meet the indicator.

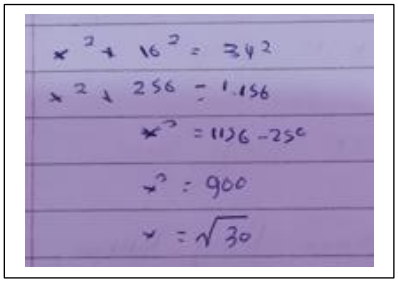

Figure 9. Example of a Student's Answer that Does Not meet the third indicator

Based on Figure 9, it was found that the student was able to identify the concept in solving problems related to Pythagorean Triples; however, the student made errors in using square root and exponent concepts. From the interview, it was revealed that the student was still confused about determining the square root value, which led to the answer shown in the image. Therefore, it is essential to provide students with a better understanding of how to work with the Pythagorean formula. As stated by [14] and [15], a solid understanding can significantly improve students' conceptual comprehension.

4. Presenting Concepts and Solving Problems in Various Mathematical Representations

In this indicator, the given question involves applying the concept of the Pythagorean Theorem in a real-life context by determining the length of a ladder. The following is the question presented to fulfill this indicator.

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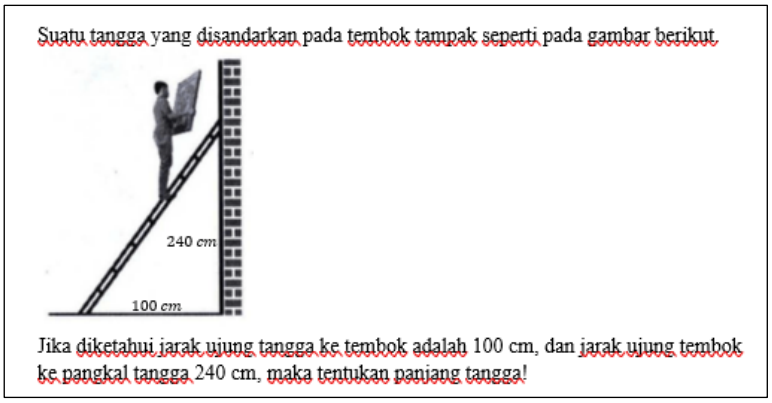

Figure 10. Question for the Indicator of Presenting Concepts and Solving Problems in Various Mathematical Representations

Based on Figure 10, the question requires students to apply the concept of the Pythagorean Theorem and represent it in real-life problem situations. From the analysis of students' answers, it was found that only two students were unable to apply the Pythagorean Theorem concept in problem-solving. This is in line with the study conducted by [16], which found that the most common type of error made by students in this type of problem is a procedural error in interpreting the situation. The following are several examples of students' answers to the problem.

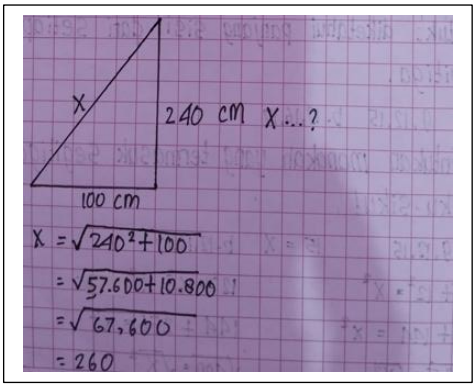

Figure 11. Example of a Student's Answer that Meets the Fourth Indicator

Based on Figure 11, it is evident that the student is able to represent the concept of the Pythagorean Theorem and apply it to real-life situations by solving the given problem. In line with the interview with the student, it was found that the student could immediately use the Pythagorean Theorem concept upon seeing the problem, thus arriving at the correct answer. For students who are not yet able to apply the Pythagorean Theorem in solving real-life problems, an example can be seen in the following figure.

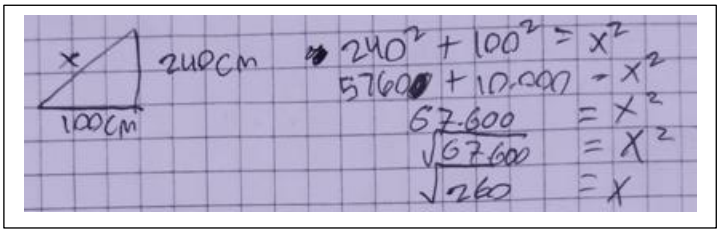

Figure 12. Example of a student's answer that does not meet the fourth indicator

Based on Figure 12, it can be seen that the student still made an error when using the square root. However, the student correctly applied the Pythagorean concept, even though they did not write the general formula of the Pythagorean Theorem in the given problem. Based on an interview with the student, it was revealed that the student is still not accustomed to writing out the Pythagorean concept beforehand, and instead, directly applies it. The student also often feels uncertain when dealing with exponents and square roots involving integers. This is in line with a study conducted by [16], which found that students made procedural errors, as also seen in this case, where the student's answer remained in squared form due to their inability to recall the correct method for extracting square roots. Based on Figure 12, it can be seen that the student still made an error when using the square root. However, the student correctly applied the Pythagorean concept, even though they did not write the general formula of the Pythagorean Theorem in the given problem. Based on an interview with the student, it was revealed that the student is still not accustomed to writing out the Pythagorean concept beforehand, and instead, directly applies it. The student also often feels uncertain when dealing with exponents and square roots involving integers. This is in line with a study conducted by [16], which found that students made procedural errors, as also seen in this case, where the student's answer remained in squared form due to their inability to recall the correct method for extracting square roots.

CONCLUSIONS AND SUGGESTIONS

Based on the results and discussion, the conclusion obtained is that the ability of eighth-grade junior high school students to conceptualize the Pythagorean Theorem, based on the National Research Council indicators, is most evident in the ability to apply and implement the concept. The indicator least mastered by students is classifying objects based on mathematical concepts, as students still experience difficulties in distinguishing between examples and non-examples in mathematics. Future researchers may explore students' conceptual understanding abilities in other mathematical topics.

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