Volume 7, Issue 1, December 2025
Journal of Research on Mathematics Instruction
DOI: https://doi.org/10.33578/jrmi.v7i1.119
Journal's Webpage: http://jrmi.ejournal.unri.ac.id

The Effect of Worksheets and Prior Knowledge on Students' Mathematical Proof Ability in the Isoperimetric Problem

Febby Ayuni Esya Putri [1]*, Denny Ivanal Hakim [2]
[1] Elementary School Teacher Education, Universitas Jambi, Muaro Jambi, 36361, INDONESIA
[2] Mathematics, Institut Teknologi Bandung, Bandung, 40132, INDONESIA

ARTICLE'S INFORMATION

Article history:

Received: Nov-17-2025

Reviewed: Dec-4-2025

Accepted: Dec-28-2025

Keywords: *Isoperimetric Problem, Mathematical Proof Ability, Prior Knowledge, Worksheet*

ABSTRACT

Mathematical proof ability is essential in mathematics learning because it supports logical reasoning, conceptual connections, and higher-order thinking. However, many high school students still struggle to construct valid mathematical proofs, highlighting the need for effective instructional strategies. One approach to strengthening proof ability is through proof-oriented tasks such as the Isoperimetric Problem, which can be explored using school-level mathematics concepts, including algebraic operations and the area and perimeter of two-dimensional figures. This study aims to examine the effects of worksheet use, students' prior knowledge, and their interaction on students' ability to construct mathematical proofs related to isoperimetric problems. A true-experimental nonequivalent control group design was employed with two second-year high school classes at MAN 2 Jambi City. The experimental group learned through worksheet-based instruction, while the control group received direct instruction. Students were categorized into high-, medium-, and low-knowledge groups. Data were analyzed using a two-way ANOVA at a 0.05 significance level. The results show that worksheet use, prior knowledge, and their interaction significantly affect students' abilities in mathematical proof.

Corresponding address:

Febby Ayuni Esya Putri,
E-mail: febbyayuniesya.p@unja.ac.id

INTRODUCTION

Mathematical proof is a tool for demonstrating the truth in mathematics by integrating logical reasoning, conceptual understanding, and problem-solving [1]–[3], which are essential for higher-order thinking skills [4], [5]. Proof skills develop gradually, and students who can build clear and coherent arguments tend to have stronger mathematical reasoning and be better prepared for future lessons [4], [5]. Furthermore, learning environments that emphasize proof help students become more skilled at judging whether mathematical claims are valid, which is an important sign of mathematical expertise[6], [7]. For these reasons, proof skills should be optimized, as they are recognized as a core process standard in mathematics education worldwide (NCTM), including in Indonesia [8]–[11].

Teachers play an essential role in developing students' proof skills by providing meaningful contexts and appropriate mathematical content for proof activities [12], [13]. One such context is the Isoperimetric Problem. The Isoperimetric Problem, or the 9th-century B.C.E. Queen Dido problem, tells about Queen Dido, who flees from her home to the North Coast of Africa after her brother kills her father [14], [15]. Queen Dido asked the ruler of the North Coast of Africa for a piece of land, which he would grant on the condition that the ground could be covered with cowhide. With that cowhide, Queen Dido found that the circle is the largest area that can be covered compared to other shapes with the same boundary [14]–[16].

Isoperimetric problems provide an authentic context for practicing proof because they require students to relate perimeter and area through deductive reasoning[17], [18]. Studies show that tasks involving area–perimeter optimization foster rigorous argumentation and are identified as effective authentic proof activities in mathematics classrooms [19]–[21]. Classic formulations, such as determining the shape of maximal area for a fixed perimeter, lead directly to the circle, a result that can be established by a variety of elegant proofs, from Steiner's geometric constructions to modern variational arguments [7], [14] . This problem looks simple, but the proof of the Isoperimetric Problem presented by experts such as Zenodorus and Steiner is difficult for students to understand and requires material beyond the high school level [14], [22]. Nevertheless, it is still possible to explore the Isoperimetric Problem using school-level mathematics by focusing on specific cases, such as triangles and quadrilaterals, summarised in the following theorems.

Theorem 1. Square has the largest area among all rectangles with the same perimeter.

Theorem 2. The area of an equilateral triangle is greater than the area of an isosceles triangle and a scalene triangle with the same perimeter

Theorem 3. The area of a square is greater than the area of a rectangle, parallelogram, trapezoid, or rhombus with the same perimeter.

Based on Theorem 1, Theorem 2, and Theorem 3, the Isoperimetric Problem is related to the minimum required school mathematics concepts, which are elementary geometry of perimeter and area of Two-Dimensional Figures, and algebraic operations. For this reason, students can prove the Isoperimetric Problem using the mathematics content they have studied in the second year of high school, such as perimeter and area of Two-Dimensional Figures, algebraic operations, and the derivative of a one-variable function. In other words, teachers can use Isoperimetric Problems as a reference for mathematical proofs to improve students' ability to prove mathematically.

For high school students, working with the Isoperimetric Problem proof is especially valuable because it brings together algebraic skills, geometric visualization, and logical thinking, all of which are central to learning geometry [23]. The Isoperimetric problem also helps students see how abstract mathematical ideas can be connected to real and observable situations, making the concepts easier to understand and more meaningful. In addition, discussing the assumptions behind the proofs, such as why certain shapes are considered optimal and how symmetry plays a role, encourages students to think more carefully about whether an argument is complete and logically sound. This kind of experience helps students improve their ability to justify solutions and explain their reasoning, areas where many students still struggle in geometry [19], [24], [25].

Because the Isoperimetric Problem is rarely encountered in secondary curricula, teachers can introduce it through a carefully designed worksheet. Worksheets serve as instructional scaffolds that sequence the stages of conjecture, exploration, and justification, helping students record known facts, organize intermediate results, and construct a coherent argument [26]–[28]. Recent studies show that worksheet-based activities can significantly improve students' problem-solving performance. However, their effectiveness largely depends on the quality of the prompts and the level of teacher guidance [29].

Before using worksheets to optimise students' proofs of the Isoperimetric Problem, teachers must assess learners' prior geometric knowledge, as they should know what students already know about the mathematical content related to the Isoperimetric Problem. Studies of prospective teachers suggest that students with strong content knowledge are better able to connect definitions, theorems, and visual representations into clear and logical arguments, resulting in stronger proofs [19]. In contrast, research on proof learning consistently shows that students with limited prior knowledge often struggle to recognize which concepts are relevant and to justify each step of their reasoning, leading to incomplete or disconnected arguments [5].

Previous research has often examined students' abilities in mathematical proof or worksheet-based instruction separately. Still, only a few studies have considered both factors together when evaluating students' ability to prove the Isoperimetric Problem, and even fewer have used quantitative methods for comparison. Reviews of proof research show that most studies focus either on how students interact with mathematical content or on how instruction is designed, without simultaneously taking into account students' prior knowledge and the specific nature of problems, such as the Isoperimetric Problem [5] . Although studies on learning in structured environments report that worksheets can support reasoning and proof, these improvements are rarely analyzed in relation to students' prior geometric knowledge [30]. Taken together, these findings reveal a clear research gap: the need for a rigorous quantitative study that examines how worksheet support and students' prior geometric knowledge jointly affect proof performance on the Isoperimetric Problem.

The explanation above is the background for researchers to conduct research aimed at three objectives. The first one is to describe the effect of the worksheet on students' ability to prove the Isoperimetric Problem. The second one is to describe the effect of students' prior knowledge on students' ability to prove the Isoperimetric Problem. The last one is to have information on the effect of the interaction between giving a worksheet and students' prior knowledge on students' ability to prove the Isoperimetric Problem.

METHODS

This study employed a true-experimental research design with a pre-test and post-test control group design. The research was conducted at MAN 2 Jambi City, Indonesia, from August 15 to August 30, 2025. Two second-year high school classes were selected through simple random sampling. One class was assigned as the experimental group, and the other as the control group.

Table 1. Research Design

Groups	Pre-test	Treatment	Post-test
Experiment	Yes	Worksheet	Yes
Control	Yes	Direct Learning	Yes

Before the treatment, both groups were given a pre-test to assess students' prior knowledge of the Isoperimetric Problem. Based on the pre-test scores, students were classified into three prior knowledge categories: high, medium, and low, using the mean and standard deviation criteria. Students whose scores were greater than the mean plus one standard deviation were categorized as high, those within one standard deviation of the mean were categorized as medium, and those whose scores were lower than the mean minus one standard deviation were categorized as low.

Table 2. Prior Knowledge Category

Criteria	Category
$x > \bar{x} + \text{stdev}$	High
$\bar{x} - \text{stdev} \leq x \leq \bar{x} + \text{stdev}$	Medium
$x < \bar{x} - \text{stdev}$	Low

Remarks:

x: students' pre-test score

$\bar{x}$: mean pre-test score

DOI: https://doi.org/10.33578/jrmi.v7i1.119
jrmi.ejournal.unri.ac.id, Online ISSN: 2715-6869

stdev: deviation standards pre-test score

After the pre-test, the experimental group received instruction using worksheet-based activities combined with cooperative learning, where students were guided to construct proofs of the Isoperimetric Problem through structured prompts. In contrast, the control group learned through direct instruction, in which the teacher explained the proof process and guided students step by step. At the end of the instructional period, both groups completed a post-test designed to assess their ability to produce mathematical proofs related to the Isoperimetric Problem.

The research instruments consisted of a pre-test and a post-test, both of which measured students' ability to construct mathematical proofs. Data from the pre-test and post-test were analyzed using a two-way analysis of variance (Two-Way ANOVA) at a significance level of 0.05 to examine the effects of worksheet use, prior knowledge, and their interaction on students' mathematical proof ability.

RESULTS AND DISCUSSION

Based on the pre-test results, the experimental class had a mean score of 40.55 (SD = 19.77), while the control class had a mean score of 48.13 (SD = 16.38). Students in both classes were categorized into high, medium, and low prior-knowledge groups based on the mean and standard deviation of their pre-test scores, as shown in Table 3. The distribution of students across prior knowledge categories is shown in Table 4.

Table 3. Categorization of Prior Knowledge

Classes	**Prior Knowledge Categories**		
	High	**Medium**	**Low**
Experiment	$x > 60.32$	$20.78 \leq x \leq 60.32$	$x < 20.78$
Control	$x > 64.51$	$31.75 \leq x \leq 64.51$	$x < 31.75$

Remarks: x is the students' pre-test score

Table 4. Percentage of Students According to Their Prior Knowledge

Classes	**Prior Knowledge Categories**		
	High	**Medium**	**Low**
Experiment	17.24% (5 Students)*	68.97% (20 Students)*	13.79% (4 Students)*
Control	17.24% (5 Students)*	68.97% (21 Students)*	13.79% (4 Students)*

*out of 29 students

Following the pre-test, the experimental class received worksheet-based instruction combined with cooperative learning, whereas the control class received direct instruction. Although both groups learned the same mathematical content, the instructional approaches differed. After the instructional intervention, students in both classes completed a post-test designed to measure their ability to construct proofs of the Isoperimetric Problem.

Before conducting the Two-Way ANOVA, tests of normality and homogeneity were performed. The normality test results indicated significance values of 0.198 for the experimental class and 0.059 for the control class, both exceeding the 0.05 threshold, suggesting that the data were normally distributed. The homogeneity test also yielded a significance value of 0.162, indicating homogeneous variances. Therefore, the assumptions for Two-Way ANOVA were satisfied.

Journal of Research on Mathematical Instruction, Vol. 7, I. 1, December 2025, 25-35
DOI: https://doi.org/10.33578/jrmi.v7i1.119
jrmi.ejournal.unri.ac.id, Online ISSN: 2715-6869

Table 5. Two-Way ANOVA Result

Source of Variation	df	Mean Square	F	Sig.
Worksheet	1	1026.286	9.945	0.003
Prior Knowledge	2	5687.432	55.114	0.000
Interaction	2	329.833	3.196	0.049
Error	52	103.195	-	-
Total	58	-	-	-

The analysis revealed a significant main effect of worksheet use ($F = 9.945$, $p = 0.003$), indicating that students who learned using worksheets demonstrated significantly higher mathematical proof ability than those who received direct instruction. This finding suggests that worksheets effectively support students in understanding and structuring proofs, particularly for unfamiliar topics such as the Isoperimetric Problem.

A significant interaction effect between worksheet use and prior knowledge was also found ($F = 3.196$, $p = 0.049$). As illustrated in Figure 1, students with high and medium prior knowledge benefited more from worksheet-based instruction than from direct instruction. However, students with low prior knowledge performed better under direct instruction than when learning with worksheets. This result suggests that students with limited prior knowledge may rely more heavily on explicit teacher explanations and may struggle to engage effectively in collaborative, worksheet-based learning without sufficient scaffolding.

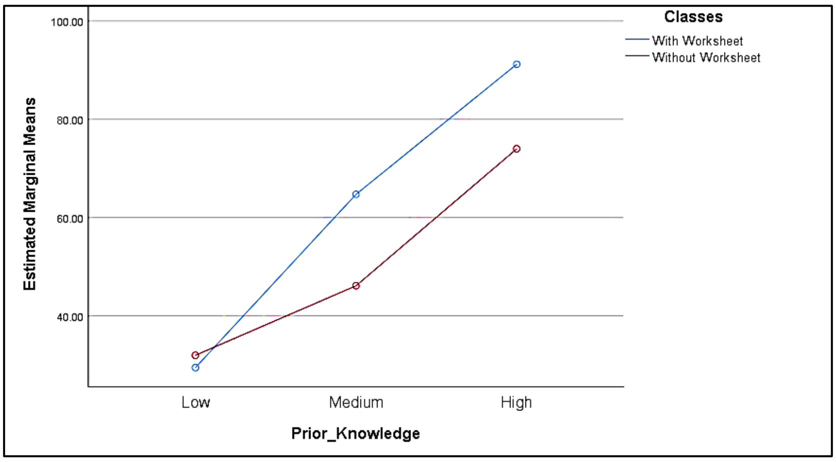

Figure 1. Interaction Variable

Overall, the findings indicate that while worksheets are effective in enhancing mathematical proof ability, their effectiveness depends on students' prior knowledge. Teachers should therefore consider students' readiness when selecting instructional strategies and provide additional guidance for students with low prior knowledge to ensure equitable learning outcomes.

The results of the Two-Way ANOVA indicate that worksheet use has a significant effect on students' ability to prove Isoperimetric Problems ($p = 0.03 < 0.05$). This finding suggests that worksheet-based instruction effectively supports students' development of mathematical proofs. Because the Isoperimetric Problem is generally unfamiliar to high school students, the structured guidance provided by the worksheets helped students understand the problem context and follow the logical steps required to construct a proof. Worksheets introduce the problem gradually, providing a structured sequence that guides students from known facts to the target theorem [31]–[33]. The step-by-step prompts embedded in the worksheets enable learners to grasp the underlying theorems more efficiently [34]. This result is consistent with previous studies showing that well-designed worksheets facilitate students' understanding of complex mathematical tasks by clarifying problem structure and reasoning processes [35]–[37].

In addition to improving conceptual understanding, the use of worksheets contributed to a more structured learning process. Students who clearly understood the Isoperimetric Problem and the stages of the proof were better able to select and apply appropriate mathematical concepts. In contrast, students in the control class, who learned through direct instruction, often struggled to identify which mathematical concepts were relevant to proving the Isoperimetric Problem, despite the teacher's explanations. This lack of clarity led to longer problem-solving time, inefficient use of classroom time, and greater dependence on teacher assistance. Worksheets served as cognitive scaffolds, guiding students step by step through the reasoning process, reducing cognitive load and helping them focus on essential relationships among concepts [38], [39]. Moreover, the structured prompts in the worksheets encouraged students to actively engage in analysis and justification rather than passively following the teacher's explanations [36], [39]. These findings support earlier research indicating that worksheets help organize students' thinking and make learning activities more systematic and manageable [40], [41].

Worksheets also played an important role in assisting teachers in monitoring students' learning progress. In the experimental class, teachers could track students' progress and identify difficulties through students' written work on the worksheets. In contrast, in the control class, teachers found it more difficult to assess students' progress, as only a few students actively articulated their ideas during the proof process. This result aligns with previous studies that emphasize that worksheets serve as an effective formative assessment tool, enabling teachers to observe students' reasoning processes more closely [36], [40].

Furthermore, the experimental class used cooperative learning in conjunction with worksheets. The findings indicate that collaboratively structured learning supported by worksheets had a greater impact on students' ability to solve Isoperimetric Problems. Collaborative learning encourages active engagement, peer discussion, and shared reasoning, allowing students to construct understanding through interaction rather than passive listening [31], [42], [43]. During group work, students in the experimental class actively exchanged ideas, discussed alternative strategies, analyzed relationships between the Isoperimetric Problem and related mathematical concepts, and formulated conclusions. These activities contributed to the development of key mathematical reasoning skills, such as identifying relationships, forming conjectures, testing proofs, and drawing logical conclusions. This result is consistent with previous research showing that proof-oriented and worked-example worksheets enhance students' proof construction and reasoning abilities [8], [29], [44].

The analysis also revealed that students' prior knowledge significantly affects their ability to prove the Isoperimetric Problem ($p = 0.007 < 0.05$). This finding supports earlier studies indicating that prior knowledge plays a crucial role in the learning process by enabling students to connect new information with existing conceptual frameworks [45]–[47]. The Isoperimetric Problem requires an understanding of prerequisite concepts such as the area and perimeter of two-dimensional figures and algebraic operations. Students who lacked mastery of these concepts encountered difficulties in constructing valid proofs, as they were unable to select or apply appropriate mathematical ideas [17], [35], [45], [48].

Students with low prior knowledge had particular difficulty in answering pre-test questions on area, perimeter, and algebraic operations. This observation is consistent with findings that students with limited prior knowledge struggle to integrate new information into their existing knowledge structures [45], [46]. To address this issue, teachers encouraged students to review prerequisite concepts before beginning the proof activities and provided brief conceptual reinforcement to reduce misconceptions during the learning process [39], [49].

Differences in problem-solving strategies were also observed across prior knowledge categories. Students with high prior knowledge employed multiple strategies to determine areas of various shapes with equal perimeters, such as comparing the areas of equilateral and isosceles triangles, or squares with other quadrilaterals. In contrast, students with medium and low prior knowledge frequently relied on inappropriate strategies, such as incorrectly applying the Pythagorean Theorem to determine heights or diagonals. These students required additional hints from the teacher to explore alternative strategies. This finding aligns with research suggesting that students with higher prior knowledge possess a broader repertoire of strategies and are better able to select appropriate approaches for solving complex problems [45], [50], [51].

The interaction effect between worksheet use and prior knowledge was also statistically significant ($p = 0.049 < 0.05$). The interaction plot (Figure 1) shows that while students with high and medium prior knowledge benefited more from worksheet-based instruction, students with low prior knowledge performed better under direct instruction. This result indicates that students with low prior knowledge may rely more on explicit teacher explanations and structured guidance than on collaborative worksheet-based activities [45], [50], [51]. When learning with worksheets, these students tended to accept their peers' solutions without fully understanding the underlying concepts, leading to weaker performance on proofs [52].

This finding reinforces the idea that instructional strategies should be selected by considering students' characteristics, learning readiness, and prior knowledge [39], [42], [50]. Although worksheets are effective in supporting proof construction, additional scaffolding or more explicit instruction may be necessary for students with low prior knowledge to ensure that they can meaningfully engage with proof-oriented tasks [45], [53]. Therefore, teachers should carefully balance worksheet-based learning and direct instruction to optimize students' abilities in mathematical proof across different levels of prior knowledge [28], [33], [54].

CONCLUSIONS AND SUGGESTIONS

Based on the results and discussion, three main conclusions can be drawn from this study. First, worksheet-based instruction has a significant effect on students' ability to construct mathematical proofs of the Isoperimetric Problem. Second, students' prior knowledge significantly influences their ability to produce mathematical proofs. Third, there is a significant interaction between worksheet use and prior knowledge, indicating that the effectiveness of instructional strategies depends on students' learning readiness.

The findings show that worksheets provide structured guidance that helps students understand the Isoperimetric Problem and follow the logical steps required for proof construction. Students with strong prior knowledge of prerequisite concepts, such as area, perimeter, and algebraic operations, demonstrate higher proof performance. Moreover, the interaction results reveal that worksheet-based learning is more effective for students with high and medium prior knowledge. In contrast, students with low prior knowledge tend to benefit more from direct instruction. These findings suggest that instructional design should consider students' characteristics, prior knowledge levels, and the nature of the mathematical content to optimize students' abilities in mathematical proof.

This study has several limitations. The intervention was conducted within a limited time frame and focused only on two-dimensional Isoperimetric Problems. Proofs of the Isoperimetric Theorem for polygons with $n \geq 5$ and applications to three-dimensional objects were not examined. Future research is recommended to explore more advanced forms of the Isoperimetric Problem, incorporate longer instructional interventions, and investigate additional scaffolding strategies for students with low prior knowledge. Despite these limitations, this study provides empirical evidence on the roles of worksheet-based instruction and prior knowledge in supporting students' construction of mathematical proofs in secondary mathematics education.

REFERENCE

[1] S. G. Krantz, *The History and Concept of Mathematical Proof*. 2007. [Online]. Available: https://www.researchgate.net/publication/250893464_The_History_and_Concept_of_Mathematical_Proof

[2] K. Weber, "Problem-Solving, Proving, and Learning: The Relationship Between Problem-Solving Processes and Learning Opportunities in the Activity of Proof Construction," *J. Math. Behav.*, vol. 24, no. 3–4, pp. 351–360, 2005, doi: https://doi.org/10.1016/j.jmathb.2005.09.005.

[3] E. J. Knuth, "Proof as a Tool for Learning Mathematics," *Math. Teach. Learn. Teach. PK-12*, vol. 95, no. 7, pp. 486–490, 2002, doi: https://doi.org/10.5951/MT.95.7.0486.

[4] G. Hanna, *Proof and Proving in Mathematics Education*. New York: Springer, 2021. doi: 10.1007/978-94-007-2129-6.

[5] G. J. Stylianides, A. J. Stylianides, and A. Moutsios, "Proof and Proving in School and University Mathematics Education Research : A Systematic Review," *ZDM – Math. Educ.*, vol. 56, no. 1, pp. 47–59, 2024, doi: 10.1007/s11858-023-01518-y.

[6] I. Garnelo and M. Liebendörfer, "Towards Design Principles for Learning Environments Based On Theorem Provers," in *Proceedings of the Fourteenth Congress of the European Society for Research in Mathematics Education*, 2025.

[7] H. Wu, "The Isoperimetric Inequality: the Algebraic Viewpoint," in *Math Handbook*, Berkeley: University of California, 1999, pp. 79–81.

[8] E. Nurlaelah, A. Pebrianti, M. Taqiyuddin, J. A. Dahlan, and D. Usdiyana, "Improving Mathematical Proof Based On Computational Thinking Components for Prospective Teachers In Abstract Algebra Courses," *Infin. J. Math. Educ.*, vol. 14, no. 1, pp. 85–108, 2025, doi: https://doi.org/10.22460/infinity.v14i1.p85-108.

[9] L. Anwar, A. Mali, and M. J. Goedhart, "The Effect of Proof Format on Reading Comprehension of Geometry Proof : The Case of Indonesian Prospective Mathematics Teachers," *EURASIA J. Math. Sci. Technol. Educ.*, vol. 17, no. 4, 2021, doi: https://doi.org/10.29333/ejmste/10782.

[10] S. H. Teoh, Rahmawati, S. Parmijt, and H. Retnawati, "Exploring Mathematical Reasoning and Proof with Indonesian Senior High School Students," *Malaysian J. Math. Sci.*, vol. 19, no. 3, pp. 837–855, 2025, doi: https://doi.org/10.47836/mjms.19.3.04.

[11] Isnani, S. B. Waluya, Dwijanto, and T. S. N. Asih, "Grow Proof Ability Student Through Mathematical Working Space Strategies," *Edelweiss Appl. Sci. Technol.*, vol. 9, no. 4, pp. 1240–1251, 2025, doi: 10.55214/25768484.v9i4.6229.

[12] D. Rahimah, Susanta, Susanto, and Utari, "The Indonesian Independent Curriculum Requires Student-Centred Teaching Approaches: Does the Teacher Guide Accompanying a Grade VIII Ministry-Published Mathematics Textbook Assist Teachers?," *Educ. Q. Rev.*, vol. 7, no. 1, 2024, doi: 10.31014/aior.1993.07.01.568.

[13] D. Isran, A. Susanta, D. Rahimah, and F. S. Syafri, "Exploring Mathematical Proofs and Solutions in Higher Education : A Case Study on Real Analysis," *J. Pendidik. Mat.*, vol. 19, no.

1, pp. 163–180, 2025, doi: https://doi.org/10.22342/jpm.v19i1.pp163-180 Exploring.
[14] A. Siegel, "A Historical Review of the Isoperimetric Theorem in 2-d, and Its Place in Elementary Plane Geometry," *Courant Inst. Math. Sci.*, pp. 1–11, 2003.
[15] V. Blåsjö, "The Isoperimetric Problem," *Am. Math. Mon.*, vol. 112, no. 6, pp. 526–566, 2005, doi: 10.1080/00029890.2005.11920227.
[16] M. Kline, *Mathematics for the Non-Mathematician*. Canada: General Publishing Company, 1967. doi: 10.1038/288037a0.
[17] Eskelson, Samuel, T. Brian, Hughes, and Elizabeth, "Maximizing Area and Perimeter Problems," *Natl. Counc. Teach. Math.*, vol. 144, no. 1, 2021, doi: https://doi.org/10.5951/MTLT.2019.0310.
[18] J. Boyle, S. Bleir, and S. Yee, "Transforming Perceptions of Proof: A Four-Part Instructional Sequence," *Natl. Counc. Teach. Math.*, vol. 4, no. 1, 2021, doi: https://doi.org/10.5951/mathteaceduc.4.1.0032.
[19] O. Buchbinder and S. Mccrone, "Preparing Prospective Secondary Teachers to Teach Mathematical Reasoning and Proof : the Case of the Role of Examples In Proving," *ZDM – Math. Educ.*, vol. 55, no. 4, pp. 779–792, 2023, doi: 10.1007/s11858-023-01493-4.
[20] Melhuish, Dawkins, and Lew, "Lessons Learned About Incorporating High-Leverage Teaching Practices in the Undergraduate Proof Classroom to Promote Authentic and Equitable Participation," *Int. J. Res. Undergrad. Math. Educ.*, vol. 10, 2022, doi: https://doi.org/10.1007/s40753-022-00200-0.
[21] M. Applebaum, "Proof by Play : Teaching the Parity Principle With Math Games and Puzzles," *Contemp. Math. Sci. Educ.*, vol. 6, no. 2, 2025, doi: https://doi.org/10.30935/conmaths/17404.
[22] T. C. Hales, "The Honeycomb Conjecture," *Arxiv*, vol. 25, no. 1, 1999, doi: https://doi.org/10.48550/arXiv.math/9906042.
[23] M. L. Winer and M. T. Battista, "Investigating Students ' Proof Reasoning : Analyzing Students ' Oral Proof Explanations and their Written Proofs in High School Geometry," *Int. Electron. J. Math. Educ.*, vol. 17, no. 2, 2022, doi: https://doi.org/10.29333/iejme/11713.
[24] I. Kontorovich, "' Find the area enclosed by ...' Parceling an Especially Robust Model of Reasoning Among First - Year Students," *Int. J. Res. Undergrad. Math. Educ.*, vol. 9, no. 1, pp. 149–172, 2023, doi: 10.1007/s40753-023-00213-3.
[25] A. M. Ali and D. I. Hakim, "Isoperimetric problems on n -sided prisms," *Hilbert J. Math. Anal.*, vol. 3, no. 1, pp. 34–41, 2024, doi: DOI: https://doi.org/10.62918/hjma.v3i1.36.
[26] S. A. Courtney and B. Armstrong, "Promoting Geometric Reasoning through Artistic Constructions," *Eur. J. STEM Educ.*, vol. 6, no. 1, pp. 1–10, 2021, doi: https://doi.org/10.20897/ejsteme/11332.
[27] R. Govender, G. Amevor, M. Education, C. Town, and R. Govender, "Instructional-Based Learning of Cyclic Quadrilateral Theorems : Making Geometric Thinking Visible and Enhancing Learners ' Spatial and Geometry Cognitions," *Pythagoras - J. Assoc. Math. Educ. South Africa*, vol. 46, no. 1, pp. 1–12, 2025, doi: https://doi.org/10.4102/ pythagoras.v46i1.812.
[28] S. Novita, P. Yunita, S. Novika, and H. S. Ruth, "Development of E-Worksheet Based on Realistic Mathematics Education to Support Mathematical Literacy Skills of Junior High School Students," *E3S Web Conf.*, 2023, doi: https://doi.org/10.1051/e3sconf/202340003006.
[29] S. A. Widodo, A. Wijayanti, M. Irfan, W. Pusporini, S. Mariah, and S. Rochmiyati, "Effects of Worksheets on Problem-Solving Skills : Meta-Analytic Studies," *Int. J. Educ. Methodol.*, vol. 9, no. 1, pp. 151–167, 2023, doi: https://doi.org/10.12973/ijem.9.1.151.
[30] M. A. K. Fraihat, A. A. Khasawneh, and A. A. Al-Barakat, "The Effect of Situated Learning Environment In Enhancing Mathematical Reasoning and Proof Among Tenth Grade Students," *EURASIA J. Math. Sci. Technol. Educ.*, vol. 18, no. 6, 2022, doi: https://doi.org/10.29333/ejmste/12088.
[31] F. Vogel, I. Kollar, F. Fischer, K. Reiss, and S. Ufer, "Adaptable Scaffolding of Mathematical

Argumentation Skills : The Role of Self - Regulation When Scaffolded with CSCL Scripts and Heuristic Worked Examples," *Int. J. Comput. Collab. Learn.*, vol. 17, pp. 39–64, 2022, doi: 10.1007/s11412-022-09363-z.

[32] Z. Ç. Gürel, *Indication of Scaffolding in Mathematical Modeling*, vol. 23, no. 7. Springer Nature Singapore, 2025. doi: 10.1007/s10763-025-10576-5.

[33] J. P. H. Kindica and R. M. Gumanoy, "Improving Academic Achievement of Students in Mathematics 10 through Contextualized Learning Worksheet," *Int. J. Res. Innov. Soc. Sci.*, vol. 9, no. 4, pp. 2873–2880, 2025, doi: 10.47772/IJRISS.

[34] S. Schukajlow, J. Kolter, and W. Blum, "Scaffolding mathematical modelling with a solution plan," *ZDM - Math. Educ.*, vol. 47, 2015, doi: https://doi.org/10.1007/s11858-015-0707-2.

[35] Y. Arni, A. O. Andira, G. Nabilla, and MURSIANI, "Improving Student Learning Outcomes in Determining the Area and Perimeter of Flat Shapes Through the Use of Drill Method," *Contin. Educ.*, vol. 5, no. 1, pp. 62–71, 2024, doi: 10.51178/ce.v5i1.1788.

[36] W. R. Tumangger, I. A. Khalil, and R. C. I. Prahmana, "The Impact of Realistic Mathematics Education-based Student Worksheet for Improving Students ' Mathematical Problem-Solving Skills Wana Rukmana Tumangger Rully Charitas Indra Prahmana *," *IndoMath*, vol. 7, no. 2, pp. 196–215, 2024.

[37] A. Yousaf, H. Moin, S. Majeed, R. Shafi, and S. Mansoor, "The Positive Impact of Introducing Modified Directed Self-Learning Using Pre – Small Group Discussion Worksheets As An Active Learning Strategy in Undergraduate Medical Education," *Med. Educ. Online*, vol. 28, no. 1, 2023, doi: 10.1080/10872981.2023.2204547.

[38] Y. Shin, J. Jung, S. Choi, and B. Jung, "The Influence of Scaffolding for Computational Thinking On Cognitive Load and Problem-Solving Skills In Collaborative Programming," *Educ. Inf. Technol.*, vol. 30, 2025, doi: https://doi.org/10.1007/s10639-024-13104-0.

[39] A. Prinz-weiß and E. Röhm, "To What Extent Can Think Sheets With Or Without Reflection Prompts Support Self-Regulated Learning From Text ?," *Metacognition Learn.*, vol. 20, no. 17, 2025, doi: https://doi.org/10.1007/s11409-025-09419-z.

[40] C. Areizaga, B. Dana, M. Cotto, and S. N. Clerjuste, *A Meta - Analysis of the Worked Examples Effect on Mathematics Performance*. Springer US, 2023. doi: 10.1007/s10648-023-09745-1.

[41] I. Tsabitah and Z. Abidin, "Problem Based Learning and Digital Worksheet : Can they sharpen student ' s problem solving skills ?," *United Int. J. Res. Technol.*, vol. 05, no. 03, pp. 100–104, 2023.

[42] C. Zhang, X. Liu, A. Arbor, K. Ziska, S. Jeon, and C. Yu, "Mathemyths : Leveraging Large Language Models to Teach Mathematical Language through Child-AI Co-Creative," *CHI Conf. Hum. Factors Comput. Syst.*, 2024, doi: 10.1145/3613904.3642647.

[43] W. Hulukati, M. R. Pautina, M. A. Lakadjo, N. Yutapratama, and S. Korompot, "Designing a Collaborative Critical Inquiry Module for Primary Education : Integrating Character Values through a Comprehensive Literature Review," in *Proceedings of the 1st International Conference on Educational Science and Teacher Education (ICESTE 2025)*, 2025. doi: 10.2991/978-2-38476-489-1.

[44] H.-Y. Hsu and E. A. Silver, "Cognitive Complexity of Geometric Proof and Geometric Calculation : A Problem - Solving Perspective," *Educ. Stud. Math.*, 2025, doi: 10.1007/s10649-025-10434-9.

[45] M. Hendi, S. Azmi, and N. M. I. Kertiyani, "Hubungan Kemampuan Awal Terhadap Kemampuan Pemecahan Masalah Matematis Siswa," *JCAR*, vol. 7, no. 3, 2025, doi: https://doi.org/10.29303/jcar.v7i3.11940.

[46] M. Muntazhimah, T. Turmudi, and S. Prabawanto, "The Relation Between Prior Knowledge and Students ' Mathematics Reflective Thinking Ability," *J. Phys. Conf. Ser.*, no. 1731, 2021, doi: 10.1088/1742-6596/1731/1/012043.

[47] E. Sudihartinih and S. Prabawanto, "Prior Knowledge of Students In Analytical Geometry In

DOI: https://doi.org/10.33578/jrmi.v7i1.119
jrmi.ejournal.unri.ac.id, Online ISSN: 2715-6869

Terms of Gender Analyzed Using the Rasch Model," *J. Phys. Conf. Ser.*, vol. 1806, no. 1, 2021, doi: 10.1088/1742-6596/1806/1/012056.

[48] F. A. E. Putri, S. Syaiful, and J. Siburian, "Kemampuan Berpikir Kritis Siswa pada Pembelajaran Online Inquiry dan Problem Based Learning Ditinjau dari Kemampuan Awal," *J. Cendekia J. Pendidik. Mat.*, vol. 5, no. 1, pp. 274–285, 2021, doi: 10.31004/cendekia.v5i1.450.

[49] D. P. Sari, S. Saputra, A. A. Ardywinata, R. A. Sukmawati, and M. N. Fadillah, "Students ' Errors and Misconceptions in Solving Fundamental Mathematics Problem," *J. Pendidik. Mat.*, vol. 17, no. 3, pp. 313–324, 2023, doi: https://doi.org/10.22342/jpm.17.3.21128.313-324.

[50] E. F. Ningsih and E. Retnowati, "Prior Knowledge in Mathematics Learning," vol. 467, no. Semantik 2019, pp. 61–66, 2020, doi: 10.2991/assehr.k.200827.118.

[51] S. Rach and S. Ufer, "Which Prior Mathematical Knowledge Is Necessary for Study Success in the University Study Entrance Phase? Results on a New Model of Knowledge Levels Based on a Reanalysis of Data from Existing Studies," *Int. J. Res. Undergrad. Math. Educ.*, vol. 6, no. 3, pp. 375–403, 2020, doi: 10.1007/s40753-020-00112-x.

[52] A. Brandão, L. Pedro, and N. Zagalo, "Teacher professional development for a future with generative artificial intelligence – An integrative literature review," *Digit. Educ. Rev.*, no. 45, pp. 151–157, 2024, doi: 10.1344/der.2024.45.151-157.

[53] N. Nabilah and S. Suyanto, "A CIPP-Based Evaluation of Critical Thinking In Science : Implementation of the Merdeka Curriculum in Mataram Public Junior High Schools," *STEM Educ.*, vol. 5, no. 4, pp. 587–616, 2025, doi: 10.3934/steme.2025028.

[54] D. Susanti and Suparman, "Design Development Work Sheet Transformation Geometry Based on Van Hiele Geogebra Assisted to Improve The Understanding of The Student Concept," in *International Summit on Science Technology and Humanity*, 2018, pp. 227–234.

BIOGRAPHY

Febby Ayuni Esya Putri

The author is a lecturer in the Primary School Teacher Education (PGSD) program at Universitas Jambi (UNJA). She completed her master's degrees in Mathematics Education at Universitas Jambi and in Mathematics Teaching at the Bandung Institute of Technology (ITB). She is a Lecturer in the Primary School Teacher Education (PGSD) Program, Universitas Jambi (UNJA). She can be contacted via email at febbyayuniesya.p@unja.ac.id.

Denny Ivanal Hakim

The author is a senior lecturer in the Mathematics Study Program at the Bandung Institute of Technology (ITB). He holds a Doctor of Philosophy in Mathematics from Tokyo Metropolitan University (2018), a Master of Science from the Bandung Institute of Technology (2013), and a Bachelor of Science from the same institution (2012). He currently serves as an Associate Professor (Lektor Kepala) and is a permanent lecturer at ITB. His academic and professional activities focus on mathematics education and research. He can be contacted via email at dhakim@math.itb.ac.id.