Analysis of Learners' Errors to Solve Problems on Three-Variable Linear Equation System Material Based on Newman Stages

Bunga Ariski 1, Yulie Arni 2
1 Universitas Riau, INDONESIA
2 SMK Muhammadiyah 2 Pekanbaru, INDONESIA

ARTICLE'S INFORMATION

Article history:
Received: Apr-05-2024
Reviewed: Jun-01-2024
Accepted: Jun-17-2024

Keywords: Analysis, Errors, Newman Stages, Three-Variable Linear Equation System

ABSTRACT
The purpose of this research is to describe the errors made by students in solving problems on the topic of the Three-Variable Linear Equation System (SPLTV). The problems are presented in the form of story problems. This research is a descriptive study with a qualitative approach. The subjects in this study were students from class X PPLG at SMK Muhammadiyah 2. A total of 6 subjects were selected from 24 students, consisting of 2 students from the upper group, two from the middle group, and two from the lower group. The selection of 6 subjects was intended to provide a diverse representation of student abilities, allowing for a more in-depth analysis of the common types of errors at each ability level. Data collection techniques were conducted using test instruments. Meanwhile, the data analysis technique in this study is a qualitative analysis based on Newman's framework to identify and classify the errors made by students in solving story problems in the SPLTV material. The results showed that most students still made mistakes in solving story problems based on the types of Newman stage errors, namely 1) Two students were wrong at the reading stage, 2) Two students were wrong at the comprehension stage, 3) Two students were wrong at the process skill stage, 4) Two students were wrong at the transformation stage, 5) One student was wrong at the encoding stage.

INTRODUCTION
Education is a field that focuses on the learning process [1]. Education is a process that is carried out to build and develop human potential [2]. In Law No.20 of 2003, education is an effort that can create a learning atmosphere and learning process so that students actively develop their potential to have religious and spiritual strength, self-control, personality, intelligence, noble character, and skills needed by themselves, society, nation and state. Mathematics learning aims to enable students to understand the concept of problem-solving, design mathematical approaches, solve problems, and interpret the solutions they find [3]. Through education, students should be given competencies that are essential for the current era, including adaptability, communication skills, reactivity, digital literacy, comprehension, critical thinking, resilience, and creativity [4]. One of the subjects learnt through education is mathematics.

Mathematics is an important subject in the academic curriculum, providing a foundation for critical thinking and problem-solving skills that are essential in various fields of study and professional endeavours. As a discipline, mathematics encourages logical reasoning, precision, and analytical thinking, equipping students with the ability to deal systematically with complex problems. The importance of mathematics extends beyond the classroom, influencing technological advances,
scientific discoveries and economic strategies. In educational settings, mathematics serves as a key tool to foster intellectual discipline and cognitive development, enabling students to understand and manipulate abstract concepts and apply these skills in real-world scenarios.

Maths is an important science in everyday life, offering tools to solve a variety of problems. Mathematics has a hand in providing a variety of abilities and attitudes needed by humans in order to live intelligently in their environment [5]. Learning mathematics aims to equip individuals with the ability to think logically, analytically, systematically, critically, and creatively. Minister of National Education Regulation No. 22 of 2006 emphasizes that mathematics education is not only for academic achievement but also to develop the ability to think complexly, innovatively and creatively. Mathematics is not only related to memorizing formulas and quick calculations but also involves the application and appropriate use of formulas to solve real-world problems. Learning mathematics is not just about understanding concepts but also involves the application of these concepts in solving problems, both in the field of mathematics and in everyday life [6].

Mathematics is an important foundation in education at all levels, including in vocational high schools (SMK). The emphasis on mathematics in SMK not only forms the basis of theoretical knowledge but also develops practical skills essential for vocational disciplines. Maths in SMK plays a crucial role in preparing students for the increasingly data-driven and analytical world of technical work. Solid mathematical skills are required in various vocational fields, such as engineering, information technology, and accounting, where students need to be able to apply mathematical concepts in real-life contexts. For example, in vocational engineering, maths is used to calculate the dimensions, durability and efficiency of structures or machines. In accounting, maths is required to manage finances and perform economic calculations. Therefore, the teaching of mathematics in SMK should focus on the practical application of mathematical concepts to bridge theory and practice.

One of the mathematics materials that vocational high school students must learn is the System of Linear Equations of Three Variables (SPLTV). In SPLTV, the usual problem is in the form of a contextual story. To solve the problem, there needs to be a stage of mathematical modelling or representing, and then the problem can be solved. Many students still have difficulty learning SPLTV material. This was revealed by [7] based on the results of his research on the analysis of errors in solving the story problem of the Three Variable Linear Equation System based on Polya's solution steps, stating that the types of errors made by students and the factors that cause errors made by students include (1) errors in understanding the problem (2) errors in making plans (3) errors in carrying out plans and (4) errors in re-examining the solutions obtained.

System of Linear Equations of Three Variables (SPLTV) is an essential element in advanced mathematics education, highlighting the complexity of algebraic manipulation and problem-solving in the context of related variables. This material is essential for developing students' ability to analyze and solve complex problems consisting of three interrelated linear equations. Mastery of SPLTV is essential for an in-depth understanding of linear algebra and its applications across a range of scientific and engineering disciplines. It allows students to interact with more complex mathematical models and provides a solid foundation for advanced studies in mathematics and related fields. Understanding SPLTV is indispensable for students as they navigate the mathematical challenges they will face in their academic and professional careers.

In teaching SPLTV, it is often presented in the form of story problems, which ask students to apply their mathematical knowledge to real situations. These story problems are designed to test students' understanding of SPLTV by challenging them to extract relevant information, translate it
into a set of linear equations, and then solve those equations to find the solution. This approach not only assesses their understanding of mathematical concepts but also enhances their abilities in problem-solving, critical thinking, and connecting abstract mathematical theories to practical applications. By solving SPLTV story problems, students learn to navigate complex scenarios and develop a broader and more applied understanding of linear algebra.

Story problems in mathematics are presented as meaningful narratives or sentences that relate to students’ everyday experiences, embedding mathematical concepts in a familiar context [8]. These problems serve not only to apply mathematical theory to practical situations but also to assess students’ understanding of the mathematical concepts they have learnt [9]. Proficiency in mathematics is therefore demonstrated through the ability to solve these story problems accurately.

Story problems in mathematics aim to develop mathematical skills and improve students’ understanding of mathematical concepts. These problems encourage students to practice deductive reasoning, recognize the relevance and application of mathematics in everyday life, and strengthen their mastery of mathematical skills and concepts [10]. However, often, in the process of solving these story problems, students experience difficulties and make mistakes.

Ulifa stated that an error is a form of deviation from the right thing, a predetermined procedure, or a deviation from an existing expectation [11]. Hamzah stated that if students are unable to solve a problem, it will result in errors in the solution process [12]. According to [13], student errors in solving problems can be a clue to finding causal factors that affect low learning achievement. Solichan argues that error analysis is an investigative effort to see, observe, know, find, understand, examine, classify, and explore various forms of deviation from what is considered correct or deviation from something that has been previously determined or agreed upon [14].

Newman's analysis can be used to determine the types of student errors in performing mathematical problems in writing [15]. By knowing the types of students' errors in performing mathematical problems, teachers can provide learning solutions so that students can be correct and accurate in communicating ideas through writing problems. Newman classifies students' errors in solving mathematics problems as follows [16].

a. Reading

The stage where learners are able to read keywords or symbols in the problem but have difficulty proceeding to the right solution step or difficulty in reading the question and writing down the information contained in the problem.

b. Comprehension

This is the stage where learners are able to read all the words in the problem but do not fully understand the overall meaning of the words, so learners have difficulty proceeding to the right solution step or do not understand what is asked in the problem.

c. Transformation

This is the stage where learners are able to understand what is asked in the problem but have difficulty recognizing the operations and steps needed to solve the problem.

d. Process Skill

This is the stage where learners have recognized the appropriate operation or procedure but do not yet understand the procedure needed to complete the operation accurately.

e. Encoding

This is the stage where learners have successfully found a solution to the problem but have difficulty in determining the correct final answer or fail to present the right answer.
The purpose of this research is to find out and describe the errors experienced by students of class X PPLG SMK Muhammadiyah 2 in solving a system of linear equations of three variables (SPLTV) based on Newman's classification of errors. By analyzing student errors, the researcher hopes that the results of this analysis can help students know the types of errors and factors that cause student errors on the topic of the System of Linear Equations of Three Variables.

**METHODS**

This research uses a qualitative approach. According to [17], qualitative research methods are research methods based on philosophy used to research scientific conditions (experiments) where the researcher acts as the main instrument. Qualitative research, according to Creswell & Creswell, is a qualitative approach to data collection, analysis, interpretation, and report writing that is different from traditional quantitative approaches [18]. Furthermore, [18] state that purposeful sampling, more open-ended data collection, text or image analysis, representation of information through pictures and tables, and personal interpretation of findings are all characteristics of qualitative methods.

The data collection technique used in this research is a written test. In this case, the test aims to determine the location of student errors based on the stages of Newman's analysis in the form of story problems about the Three Variable Linear Equation System. The subject of this research was conducted on class X PPLG students at SMK Muhammadiyah 2 Pekanbaru. In this study, purposeful sampling was utilized to select the subjects. Specifically, six students from class X PPLG at SMK Muhammadiyah 2 were chosen to represent different levels of ability: two from the upper ability group, two from the middle ability group, and two from the lower ability group. This sampling strategy is justified by the need to understand the range of common errors across different ability levels, which is crucial for a nuanced analysis of the challenges students face in solving Three-Variable Linear Equation System problems. Purposeful sampling allows for a more focused examination of particular phenomena within the data, aligning with the goal of qualitative research to explore deeper insights into participant experiences and perspectives. By selecting students from distinct ability groups, the study aims to capture a broad spectrum of cognitive strategies, misunderstandings, and learning difficulties, thereby providing a comprehensive understanding of the educational issue at hand. This approach is consistent with the qualitative method's emphasis on detailed situational analysis and contributes significantly to the interpretative depth of the study's findings.

Error analysis based on Newman stages, namely: 1) Reading, 2) Understanding, 3) Transformation, 4) Process Skills, 5) Encoding. The following is the formula for measuring the percentage of student errors [19].

\[
P = \frac{n}{N} \times 100\%
\]

**Description**

P : Percentage of student answer errors
n : The number of students who made mistakes
N : The number of students who worked on the problem

To see the category of student errors, the data results were analyzed based on Newman's stages on the following error indicators:

a. Reading

The error indicator at this stage is unable to read the words proposed in the problem.

b. Comprehension

The error indicators at this stage are: 1) Wrongly writing what is known from the problem; 2)
Writing down what is known and asked but not continuing the process; 3) Writing down what is known and asked but not meaningful (unclear); 4) Writing down what is known and asked but missing important information.

c. Transformation

The error indicators at this stage are: 1) Not transforming the information in the problem into a linear equation; 2) Transforming the information in the problem into a linear equation but inaccurately; 3) Already transforming the information in the problem but not writing the complete information (in tens, hundreds, or thousands).

d. Process Skill

The error indicators at this stage are 1) Error in computation; 2) Unable to continue the completion procedure (stuck); 3) Continue the computation process, but it is incorrect because there is an algebraic concept error; 4) Careless in the calculation process.

e. Encoding

The error indicators at this stage are 1) Writing notation (negative sign, symbol, equal sign, etc.); 2) Not writing variables/units; 3) Incorrect use of units.

RESULTS AND DISCUSSION

Based on the results of students' work, the results of the analysis of error types based on Newman's theory can be seen in Table 1 below.

<table>
<thead>
<tr>
<th>Error Indicator</th>
<th>Subjects Who Make Errors</th>
<th>Number (Students)</th>
</tr>
</thead>
<tbody>
<tr>
<td>Reading</td>
<td>S-5, S-6</td>
<td>2</td>
</tr>
<tr>
<td>Comprehension</td>
<td>S-5, S-6</td>
<td>2</td>
</tr>
<tr>
<td>Transformation</td>
<td>S-5, S-6</td>
<td>2</td>
</tr>
<tr>
<td>Process Skill</td>
<td>S-3, S-5, S-6</td>
<td>3</td>
</tr>
<tr>
<td>Encoding</td>
<td>S-5, S-6</td>
<td>2</td>
</tr>
</tbody>
</table>

Notes:
S1, S2 = High ability learners
S3, S4 = Middle ability learners
S5, S6 = Low ability learners

The following is the data on students' errors in solving the Three Variable Linear Equation System problem.

a. Reading

The stage where students are able to read keywords or symbols in the problem but have difficulty proceeding to the right solution steps or difficulty in reading questions and writing down information contained in the problem. At this stage, two learners made mistakes. The following are examples of learners' errors at the reading stage.

Figure 1. Learners' errors at the reading stage

Figure 1 shows that the error of S-5 learners is that they cannot read the question and write down what information is contained in the problem. This is in line with research conducted by [20], which concluded that students make mistakes at the stage of reading the information written in the problem because students are less careful and hasty in reading the problem, resulting in students having difficulty in understanding the problems contained in the problem.
b. Comprehension

This is the stage where students are able to read all the words in the problem but do not fully understand the overall meaning of the words, so they do not understand what is asked of them in the problem. At this stage, two learners made mistakes. The following are examples of learners' errors at the Comprehension stage.

![Figure 2: Learners' Errors at the Comprehension Stage](image)

Figure 2. Learners' Errors at the Comprehension Stage

Figure 2 above shows the error of S-6 students in writing what is known and asking the same as the problem but not continuing the process. This is in line with research conducted by [21], which concluded that students' mistakes when answering/working on problems usually lie in understanding; namely, students are not used to recording known and questionable clues in the problem. This results in students not being able to continue to the next process.

c. Transformation

This is the stage where learners are able to understand what is asked in the problem but have difficulty recognizing the operations and steps needed to solve the problem. At this stage, two learners made mistakes. The following are examples of learners' errors at the transformation stage.

![Figure 3: Learner Errors at the Transformation Stage](image)

Figure 3. Learner Errors at the Transformation Stage

Figure 3 above shows that the mistakes of the learners above S-3 are that they have changed the information in the problem but did not write the information completely (in tens, hundreds, or thousands). Then, the writing on \( 1x \) is not correct. It should be \( x \) only. This is in line with research conducted by [22], which concluded that students were unable to translate problem sentences into mathematical sentences (models).

d. Process Skill

This is the stage where students have recognized the appropriate operation or procedure but do not yet understand the procedure needed to complete the operation accurately. At this stage, three learners made mistakes. The following are examples of learners' errors at the Process Skill stage.
Figure 4. Learner Errors at the Process Skill Stage

Figure 4 above shows the error of S-3 learners, which is that students cannot continue the process of solving procedures (stuck). This is in line with research conducted by [23], stating that processing errors or process skills usually occur because students are confused or do not even know the next step to choose as the right problem-solving.

e. Encoding

This is the stage where students have succeeded in finding solutions to problems but have difficulty in determining the correct final answer or failing to present the right answer. At this stage, two learners made mistakes. The following are examples of learners' errors at the Encoding stage.

Figure 5. Learner errors at the Encoding stage

Figure 5 above shows the error of S-6 learners, which is that they cannot write the answer to the problem and are wrong in writing. This is in line with research conducted by [24], who concluded that the final writing error is not correct in concluding the final answer that is appropriate and on target with the context of the question in the question.

Overall, the results of this study reveal that the errors made by students at each stage of the mathematical problem-solving process reflect significant obstacles in the ongoing understanding and application of mathematical concepts. For instance, errors in the reading and comprehension stages indicate weaknesses in the fundamental mathematical literacy that is crucial for advanced mathematical learning. Consistent with findings from [20] and [21], this study suggests that the inability to understand and transform information from verbal to mathematical form effectively can impede deeper and more comprehensive mathematical learning processes. Furthermore, errors at the process skills and encoding stages, as described by [23] and [24], not only reduce short-term problem-solving effectiveness but also potentially hinder the development of more complex and creative problem-solving abilities in the future.

Given the long-term consequences of these errors, it is crucial to integrate interventions aimed at strengthening literacy and procedural skills in the mathematics curriculum. This could include the use of more diversified learning strategies, such as problem-based learning and the use of manipulatives that allow students to explore mathematical concepts in more interactive and practical ways. Through such approaches, students could better internalize the processes needed to comprehend, transform, and solve mathematical problems, thereby strengthening their foundation for advanced learning and practical applications in daily and professional life.
By comparing the results of this study with previous studies, it is evident that although there are many similarities in the types of errors reported, the level of impact and long-term implications of these errors can vary greatly depending on the educational context and implemented interventions. Therefore, further research is needed to explore how specific educational interventions can be adapted to different learning environments to reduce the prevalence of these errors and improve long-term learning outcomes for students.

CONCLUSIONS AND SUGGESTIONS

Based on the results of the study, it can be concluded that the number of students who make mistakes in solving story problems based on the Newman stage error type, namely 1) Two lower group students were wrong at the reading stage, 2) Two lower group students were wrong at the comprehension stage, 3) Two lower group students were wrong at the transformation stage, 4) Two lower group students and one middle group student were wrong at the Process Skill stage, 5) Two lower group students were wrong at the Encoding stage.

Based on the research results that have been described, several strategies are needed to improve students' understanding and skills in solving the System of Linear Equations of Three Variables (SPLTV). This is done to overcome errors in solving the System of Linear Equations of Three Variables (SPLTV). Some suggestions that the author can give are, first, related to errors at the reading stage, and it is very important to emphasize to students the importance of reading problems carefully and carefully. Approaches that can be applied include intensive reading exercises where students are invited to identify and highlight keywords and important information in the problem before trying to solve it. This exercise can help students become more thorough and reduce errors due to rushing or unclear understanding of the problem.

At the comprehension stage, students need to be helped to not only read but also really understand what is asked of them in the problem. This can be reinforced by using interactive class discussions where students are invited to analyze and debate each segment of the SPLTV problem. This kind of discussion can clarify the context of the problem and promote a deeper understanding of the concepts discussed. This method also allows students to clarify doubts and confirm their understanding of the problem before starting the solution process.

Errors in the transformation stage indicate that students often struggle to transform the problem information into a solutive mathematical model. To overcome this, more practice is needed in determining problem-solving steps and converting word problems into mathematical expressions. This practice can be enhanced through the use of varied problems, ranging from simple to complex so that students can develop their transformation skills gradually.

In the process skills stage, it is important to ensure that students understand the flow and procedures required to complete mathematical operations. Errors at this stage are often caused by confusion or uncertainty about what step to take next. Therefore, students need to be given extensive and in-depth practice in the problem-solving process, which will help them understand the sequence and logic behind each step.

Finally, at the encoding stage, the focus should be on improving students' ability to present correct and logical answers. Errors at this stage often occur due to a lack of clarity or precision in determining the final answer. Practice in writing answers that are precise and appropriate to the question can help students develop this skill. Adopting a structured and systematic approach in the problem-solving process will help students present accurate and effective answers.
To further enhance the effectiveness of instruction in addressing the errors students make in solving the System of Linear Equations of Three Variables (SPLTV), teachers can adopt several targeted approaches. In the reading stage, incorporating a "problem-solving checklist" for students to use while analyzing the problem can help in identifying key terms and crucial information, fostering a more thorough comprehension before proceeding to problem-solving. During the comprehension stage, employing a "think-aloud" method where teachers solve problems while verbalizing their thought processes can illustrate effective problem-breakdown techniques. This approach can be complemented by peer teaching sessions that promote active learning and mutual understanding among students.

For the transformation stage, integrating technology such as software that converts word problems into mathematical equations can demystify the process of forming solvable mathematical models. Additionally, providing step-by-step guides can assist students in learning how to effectively transition from a textual problem statement to a mathematical equation. In the process skills stage, simulation activities that allow students to practice the steps of mathematical operations in a controlled setting can solidify their understanding and confidence in handling mathematical procedures.

Lastly, in the encoding stage, regular targeted practice sessions that focus on refining how answers are written and critiquing sample answers can improve students' precision and accuracy in presenting their solutions. Implementing formative assessments that evaluate both the process and the final answers can provide students with specific feedback at each problem-solving stage, thereby helping them to recognize and correct their mistakes more effectively. These strategies collectively aim to provide a comprehensive approach to teaching that addresses each identified error stage while fostering a deeper mathematical understanding and problem-solving ability among students.

REFERENCE


BUNGA ARISKI
Bunga Ariski is currently studying in the Mathematics Education Program at Universitas Riau. Email: bunga.ariski6121@student.unri.ac.id

YULIE ARNI
Mathematics teacher in SMK Muhammadiyah 2 Pekanbaru.